A NOVEL APPROACH OF MULTIPLYING THREE NUMBERS NEARING DIFFERENT BASES

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ABSTRACT

Multiplication is one of the most fundamental operations in mathematics. Vedic maths is one such science which permits to think for different ways to solve a mathematical problem. Currently, there are few techniques for multiplication of three numbers e.g. product of three numbers near a base, general method for three numbers.

In this paper, we present a novel approach to perform multiplication of three numbers close to different bases (e.g. multiplying 205*694*893). The technique makes use of the algebraic equation in solving three numbers near different bases which can be considered as a special case for multiplication. Additionally, the technique can be applied successfully on three number near different bases (e.g. 34*51*69 or 2006*5993*7998). Different examples will be provided along with proof of the derived approach to facilitate people in learning the new approach.

INTRODUCTION

Conventional method of performing multiplication is quite cumbersome and increases with complexity as the numbers participating in the calculation rises. In contrast, there are various ways to perform multiplication in Vedic Mathematics which are quite easy to remember and implement. Moreover, there are some special cases which can be used to solve multiplication very quickly e.g. multiplication of 2 numbers near 100, 1000; squaring of number near a base and so on. There is always a need to have a quite optimized technique in terms of calculating mathematical operations in a quickest way.

The paper is structured as follows: in next section a novel approach for solving three numbers is proposed nearing different bases. The approach will be followed by few examples. Moreover, algebraic proof will be given for the verification of the steps used for solving multiplication of three numbers. After this, usage of proportional formula will be combined to help solve different set of numbers. Towards the end, conclusion and future scope will be discussed.

PROPOSED METHODOLOGY

The proposed methodology will contain the following steps for multiplying numbers.

Step 1. Represent the values in a matrix

Let p, q, r are the values representing different bases near a common base e.g. $3,5,4$ represent the p, q, r values for 300, 500 and 400 bases respectively (for common base 100). Moreover, a, b and c can be considered as the deficiencies or excesses of numbers from these different bases. E.g. 304 have excess of 4 over 300 and can be represented in the form $(p \;\; a)$ as $(3 \;\; 4)$.

Let the three numbers under multiplication be 201, 302 and 504. These can be represented in a matrix form as shown below

$$\begin{pmatrix} p & a \\ q & b \\ r & c \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 3 & 2 \\ 5 & 4 \end{pmatrix}$$

Step 2. There are 4 parts to the solution that needs to be calculated as shown in Fig. 1. If there are any carry forward from the previous then it needs to be adjusted properly. Moreover, negative numbers needs to be converted before calculating final solution. The number of digits in each part depends upon on the common base. Common base having values 10, 100 and 1000 will contain 1, 2 and 3 digits in each part of the solution respectively.

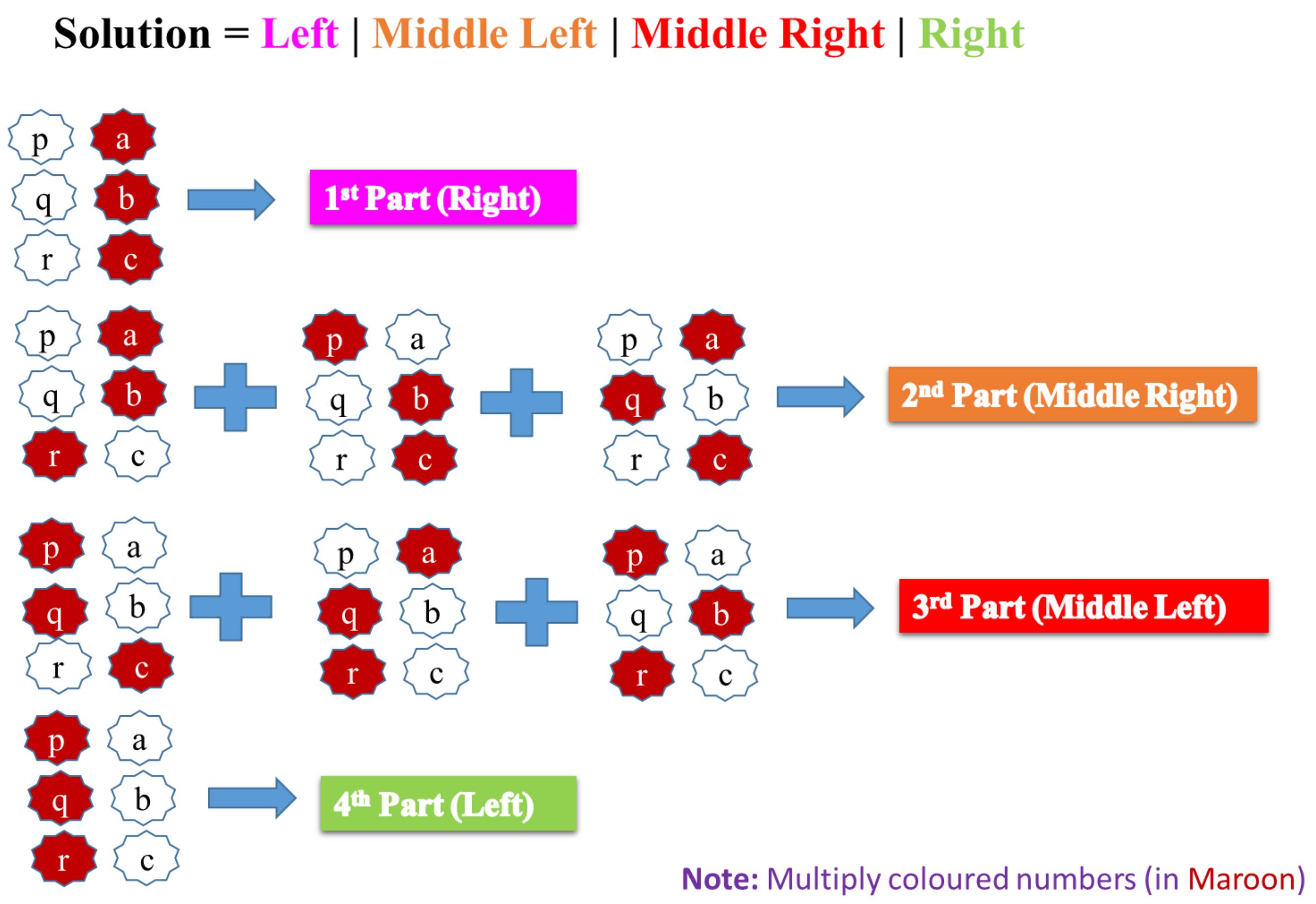

Fig. 1 Steps to multiply three number near different bases.

EXAMPLES

1) Solve $201 \times 302 \times 504$?

➔ Step 1) Represent the numbers in matrix form as shown below

$$\begin{pmatrix} 200+ & 1 \\ 300+ & 2 \\ 500+ & 4 \end{pmatrix} \Rightarrow \begin{pmatrix} 2 & 1 \\ 3 & 2 \\ 5 & 4 \end{pmatrix}$$

➔ Step 2) Calculate the four different parts as mentioned below
Right = (1*2*4) = 08 (as 2 digits are allocated)
Middle Right = (1*2*5) + (1*4*3) + (2*4*2) = 38
Middle Left = (2*3*4) + (2*5*2) + (3*5*1) = 59
Left = 2*3*5 = 30
The solution is then 30|59|38|08 or simply **30593808** which is the answer

2) Solve $297 \times 499 \times 696$?

➔ Step 1) Represent the numbers in matrix form as shown below

$$\begin{pmatrix} 300- & 3 \\ 500- & 1 \\ 700- & 4 \end{pmatrix} \Rightarrow \begin{pmatrix} 3 & -3 \\ 5 & -1 \\ 7 & -4 \end{pmatrix}$$

➔ Step 2) Calculate the four different parts as mentioned below
Right = (-3*-1*-4) = -12
Middle Right = (-3*-1*7) + (-1*-4*3) + (-3*-4*5) = 93
Middle Left = (3*5*-4) + (5*7*-3) + (3*7*-1) = -186
Left = 3*5*7 = 105

The solution is then 105|-186|93|-12
In this example, there are bar values which need to be converted/adjusted.
Therefore, it will become 103|14|92|88 or simply **103149288** which is the answer.

3) Solve $1003 \times 2995 \times 5992$?

➔ Step 1) Represent the numbers in matrix form as shown below.
In this case, common base is 1000 and therefore no. of digits in each part will be 3.

$$\begin{pmatrix} 1000+ & 3 \\ 3000- & 5 \\ 6000- & 8 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & +3 \\ 3 & -5 \\ 6 & -8 \end{pmatrix}$$

➔ Step 2) Calculate the four different parts as mentioned below
Right = (3*-5*-8) = 120
Middle Right = (3*-5*6) + (-5*-8*1) + (3*-8*3) = -122
Middle Left = (1*3*-8) + (3*6*3) + (1*6*-5) = 000
Left = 1*3*6 = 018

The solution is then 018|000|-122|120
In this example, there is a bar value -122 which needs to be converted/adjusted.
Therefore, it will become 017|999|878|120 or simply **17999878120** which is the answer.

ALGEBRAIC PROOF

$$(p+a)(q+b)(r+c) = (pq+aq+pb+ab)(r+c)$$

$$= pqr+aqr+ bpr+cpq+abr+bcp+acq+abc$$

$$= pqr+(aqr+ bpr+cpq)+(abr+bcp+acq)+abc$$

$$\left\{\begin{matrix} pqr \\ (aqr+ bpr+cpq) \\ (abr+bcp+acq) \\ abc \end{matrix}\right. \Rightarrow \left\{\begin{matrix} Left\ part \\ Left\ middle\ part \\ Right\ middle\ part \\ Right \end{matrix}\right.$$

Let the common base is 100 and p,q,r representing the different bases near the common base and a,b,c representing excess or deficient values, then pqr will be multiple of 100,00,00, whereas $(aqr+ bpr+cpq)$ will always be multiple of 100,00 and $(abr+bcp+acq)$ will be multiple of 100.

USAGE OF PROPORTIONALITY

The technique can be further enhanced with the use of proportionality formula of Vedic mathematics. Doubling and halving are the two techniques which comes under proportionality formula. In our technique there will be additional step to double one number and half the other number under consideration. Well, that can be illustrated with the help of an example.

1) Solve $148 \times 203 \times 796$?
   - ➔ Step 1) Double 148 to get 296 and half 796 to get 398.
     So, the original problem becomes solving $296 \times 203 \times 398$
   - ➔ Step 2) Represent the numbers in matrix form as shown below

$$\begin{pmatrix} 300- & 4 \\ 200+ & 3 \\ 400- & 2 \end{pmatrix} \Rightarrow \begin{pmatrix} 3 & -4 \\ 2 & 3 \\ 4 & -2 \end{pmatrix}$$

   - ➔ Step 3) Calculate the four different parts as mentioned below
     Right = (-4*3*-2) = 24
     Middle Right = (-4*3*4) + (3*-2*3) + (-4*-2*2) = -50
     Middle Left = (3*2*-2) + (2*4*-4) + (3*4*3) = -08
     Left = 3*2*4 = 24

     The solution is then 24|-08|-50|24, removing the bar numbers it becomes
     23|91|50|24 or simply **23915024** which is the answer.

CONCLUSION AND FUTURE WORK

We have seen that how we can use the proposed approach to solve some of the complex multiplication of three numbers at different bases using a simple two step approach. As compared to general method of multiplication of three number using Vedic Mathematics, this is quite easy to understand and use. Moreover, we can make use of doubling and halving formula to make the numbers aligned to their different bases and then can apply this algorithm successfully. The algorithm makes use of common base for three numbers under multiplication. Research needs to be done to allow decimal multiplication of three numbers using similar approach. Moreover, we can exploit other techniques from proportionately apart from doubling and halving to change the numbers under multiplication to their near bases. Another area of research will be to extend the algorithm to get rid of the common base for multiplying three numbers. Lastly, to extend the algorithm to multiplication of four numbers near different bases sharing common base as we did in this research paper.

REFERENCE

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