# Starring at the 



$2^{\wedge}$ Online Vedic Mathematics Conference<br>12th - 13th March 2016<br>By Giuliano Mandotti

## Abstract

The VM has one of the most powerful features that a student and a person generally may ask: the ability to develop the mental flexibility to find new solutions, simple and effective to solve the problems.

Sometimes you have to solve everyday problems in which we could use a little known feature of what for me is the magic number, the 9 .

For example if you need to find the difference 21-12 or 53-35, etc. (numbers whose digits are reversed) the typical approach would be to use the calculation method learned in the traditional curriculum of mathematics, which involves removal of figures from the units taking account of carryovers. Sometimes these calculations are simple, sometimes a bit less.

If instead we face this calculation according to the approach of the VM we discover a method to solve the problem very simple and effective, repeatable and structured according to a precise pattern and above all without the use of calculators.

The pattern I found is: starring at the 9 . Compute the difference between the digits of the numbers, multiply the result for unsigned 9 . You'll get the final result.

For example: 53-35 $=(5-3) * 9=18 ; 21-12=(2-1) * 9=9$.
My paper describes in detail this my small discovery with practical examples for subtraction of numbers 2 and 3 -digit number, where also appears a link with the formula VM for multiplication of numbers to 11. Examples studies and also for the sum, multiplication and division of these particular numbers that I called mirror numbers.

## Introduction

The number 9 encompasses many secrets and many mysteries of nature and the language of God. I have devoted much time to study and try to understand the intimate bond between the numbers and our lives.

The ancient Chinese and ancient Hindu sages had already realized the importance and perhaps they had already understood the meaning. I continue my research and found out that i'm not the only one.

All over the world there are many people who in addition to the charm of mathematics want to understand the secret. The study of numbers, I will continue to do, involves experimentation with the basic elements of our number system, the digits 1 to 9 and zero. Nine is the figure that fascinates me and it never ceases to amaze me and other people who study it.

They have been discovered many "capacity" of the digit 9, and what I propose is one that I personally discovered and that, after a thorough search, I believe has not yet been mentioned by anyone. I discovered playing with numbers and with simple operations between those numbers until I found some consistency between the numbers used and the results that I obtained gradually.

This report aims to be first an annotated summary of the discovery, so you can use it both in calculations and in the reasoning avoiding the use of calculators, computers or mobile phones; also it wants to be an incentive to have fun and manipulate numbers without fear, confident to know them and use them like any other language you already know and master.

## Background and Original Issue

In everyday life we constantly are faced with small calculations whose results we need to take more or less important decisions.

Math translates into a language, the language of the mind, the various possibilities, the options of choice we have to enable us to make the decision more useful for us.
One day 6-7 years ago now, I found myself needing to do some quick calculations to determine the size available in the small apartment in which I lived with my partner Laura for an armchair. Among the various calculations I had to make the difference between 83 cm and 38 cm .

To resolve this little problem with the traditional system I would need a sheet and a pen or dwell for a moment to do the mental subtraction 83-38 since the calculation expected a carry and, in addition to being in uncomfortable position, I was holding the meter to make measurements.

My mind immediately tried a more efficient method, which is also taken up by the VM system (the sutra by addiction and by subtraction) for the simple fact that it is common sense, intuitive and simple then mechanical subtraction digit by digit.

I had already discovered the VM system but I had not yet had time to study it, so I did not use it. I have nevertheless adopted a more effective strategy of mechanical subtract the figures starting from right to the left: I added 2 to 38 , subtracting the 40 obtained from 83 and added the 2 to obtain the final score now 45 , noting that it was a multiple 9 , but everything had stopped there.

Then when we went to buy the chair, again to decide which was better suited as a function of the spaces at our disposal I had to do a similar calculation: $54-45$, obtaining 9.

As you may have noticed the 45 was the result of the first calculation, and it was then that I connected the calculations and results, noting the curious presence of the mirror numbers and the resultant 9-drawn.

I wondered if it was just a coincidence, but we know that mathematics, as our mind, has its own structure.

Since that time, just I returned home, I took a sheet of paper and started to do other calculations with mirror numbers, coming to discover the pattern for the difference between 2-digit numbers. I started to investigate and study the Vedic mathematics and after certification as a VM Advanced Teacher I took that job to develop it in hand.

I found the pattern present in the difference between 3-digit numbers, which contextualized in the VM system now takes a broader sense considering the fact that the calculation is always maintained simple using another VM pattern, that of multiplication by 11 . We'll see later in the document.

## VM Context and VM Solution

The VM has given meaning to my little discovery, as well as giving a deeper meaning to the concept underlying mathematics itself.

In my journey of life I was not surprised to have met the VM because it is closely connected with the language of the mind and the language of God, and because it leads people to get in touch with themselves through the mind following the most simple and intuitive way, there are no efforts.

The VM allows us to leverage the power of our mind to make the best decisions for us at any given time, it allows us to increase our self-confidence and learn more easily.

I said that it is no coincidence that I met the VM because I was starting a major personal and spiritual growth work that allowed me by the way of becoming a Life Coach.

We now see the pattern of 2-digit numbers mirror gradually, starting with the traditional method, then passing through the simplification of the calculation strategy that I adopted without knowing the VM until you get to the VM pattern to fix the 9 .

Imagine having to calculate the difference between the following numbers:

$$
83-38=45
$$

The method of calculation that we have always used is the difference digit by digit, using the carry as follows:

$$
8 \Rightarrow \odot 3
$$

$38=$

Some might think of an alternative method, strategically faster, effective and easy to apply, based on the dissociative properties of addition and subtraction, starting from the reasoning of what you need to get to the smallest number to largest:

## 38 becomes: $38+2$-2 -> $40-2$ <br> 83 becomes: $80+3$-> 83

our calculation becomes:

$$
80+3-(40-2)=80-40+3-(-2)=40+5=45
$$

This strategy is already showing a more flexible use of the mind in the search path of the solution to the problem.

I want to add another simple strategy, effective and fast, very useful especially for larger numbers.

## a) Starring at the 9: 2-digits Mirror Numbers

It is based on a property of 9 which intervenes when the two involved numbers use the same digit in reversed order, then reversed in weight according to our decimal system based on the position and we call them mirror numbers.

Therefore the application of the strategy is the following:

$$
83-38=9 * 5=45
$$

where $9 *$ is fixed, that's why I called the pattern "staring at 9 ", and where 5 is the result of the difference between the digits of the two numbers, namely:

$$
8-3=5
$$

This strategy remains valid also in the case in which the difference between the two numbers was reversed as magnitude, namely:

$$
38-83=-45
$$

So we can generalize considering the modular difference:

$$
|38-83|=9 *(|3-8|)
$$

Now we apply the strategy used for the mirror two-digit numbers to other examples to become familiar with the method:

$$
73-37=9 *(7-3)=36
$$

Have the foresight to add the minus sign in the case where the first number is less than the second. For the rest it comes to applying the multiplication table of 9 with the result of the difference between the single digits of one of the two numbers involved in the transaction.

$$
\begin{array}{r}
21-12=9 *(2-1)=9 \\
53-35=9 *(5-3)=18 \\
74-47=9 *(7-4)=27 \\
62-26=9 *(6-2)=36 \\
61-16=9 *(6-1)=45 \\
82-28=9 *(8-2)=54 \\
92-29=9 *(9-2)=63 \\
91-19=9 *(9-1)=72 \\
90-09=9 *(9-0)=81 \\
100-10=100-010=9 *(10-0)=90
\end{array}
$$

## b) Starring at the 9: 3-digits Mirror Numbers

As for the case discussed in the previous paragraph I started with a practical example and tried the pattern, knowing now that I can star the 9.
I report you directly some calculation examples in which I have already starred the 9 . From these examples is easily seen the pattern for 3-digits mirror numbers.

$$
\begin{gathered}
201-102=99=9 * 11 \\
543-345=198=9 * 22 \\
421-124=297=9 * 33 \\
925-529=396=9 * 44 \\
763-367=396=9 * 44 \\
692-296=396=9 * 44 \\
863-368=495=9 * 55 \\
872-278=594=9 * 66 \\
942-249=693=9 * 77 \\
951-159=792=9 * 88 \\
960-069=891=9 * 99
\end{gathered}
$$

Let's see in detail how the strategy works starting with an example:

$$
863-368=495
$$

The resulti s equal to:

$$
9 *[11 *(8-3)]
$$

where 8 and 3 are respectively the first and last digit of the given number knowing that both numbers have the same digits in reversed order.

Obviously the calculations are now slightly more elaborate than in the case of two-digit mirror numbers because in addition to the difference between the digit appears the multiplication by 11, quite easy but not trivial for all and then the result of this multiplication is multiplied in turn by 9 , which may make it look a complicated calculation.

In fact it is not complicated because just exploit the fast multiplication by 11 as we learned in the VM system. We try to use it to make this calculation:

$$
942-249=9 *[11 *(9-2)]=9 * 77=693
$$

To take advantage of rapid multiplication by 11 we decompose the calculation as follows:

$$
942-249=9 *[11 *(9-2)]=9 * 11 * 7=11 * 9 * 7=11 * 63=693 .
$$

The 693 was obtained by applying the fast multiplication by 11 , or is the result of $63 * 11$ in the VM that method is thus obtained:

$$
6 \quad 6+3 \quad 3=693
$$

The general rule is that the result of the multiplication of a 2 -digit number by 11 is given by the first digit ( 6 ), by the sum of two digits $(6+3=9)$, by the second digit (3).

Special case of this pattern, which is already discovered and proven by many people on the internet is the magic number198, which now makes sense because we see that the difference between the first and the last digit is always equal to 2 , so the result will be always equal to:

$$
9 *[11 * 2]=9 * 22=18 * 11=198
$$

Some examples for the magic number 198:

$$
\begin{aligned}
& 987-789=198 \\
& 876-678=198 \\
& 765-567=198 \\
& 654-456=198 \\
& 543-345=198 \\
& 432-234=198 \\
& 321-123=198 \\
& 210-012=198
\end{aligned}
$$

## Algebraic Contextualization

How do you explain this property of 9 ?
After some research I found the mathematical principle that defines and creates a broader context: the modular arithmetic.

It is a system of arithmetic for integers, where numbers will "wrap on themselves" every time they reach the multiples of a given number $n$ said module.

Such arithmetic was introduced by C.F. Gauss in his treatise Disquisitiones Arithmeticae, published in 1801.

Modular arithmetic is based on the concept of congruence modulo $n$.

Given 3 integers $a, b, n$, with $n \neq 0$, we say that $a$ and $b$ are congruent modulo $n$ if their difference is a multiple of $n$.
in this case we write: $\mathrm{a} \equiv \mathrm{b}(\bmod \mathrm{n})$, and we say that $a$ is congruent $b$ modulo $n$. For example: $38 \equiv 14(\bmod 12)$, since $38-14=24$, which is a multiple of 12 .

So if we analyze the mirror numbers in modular arithmetic context, we look at their congruence module 9 .

All numbers are matching mirror module 9 .

For example, $83-38=45=5 * 9$
$83 \equiv 38(\bmod 9)$

## Insights

Following are the insights that I'm still doing on the pattern "Staring at the 9 ", trying to explore both vertically and horizontally mathematical spaces, namely looking for pattern extensions to the operation of subtraction with more digits, or pattern extensions to other operations: addition, multiplication and division.

To date I have found immediate inspiration only in the addiction of 2 and 3-digit mirror numbers.

## a) 4-digit Mirror Numbers Subtraction

$$
\begin{aligned}
& 8643-3468=5175 / 9=575 \\
& 7293-3927=3366 / 9=374 \\
& 3541-1453=2088 / 9=232
\end{aligned}
$$

I do not see, for now, obvious patterns related to 9 . I encourage you to help me in the search.

## b) 2/3 digit Mirror Numbers Addiction

However interesting to analyze what happens to the sum of mirror numbers:

$$
\begin{aligned}
& 16+61=(1+6)^{*} 11=7 * 11=77 \\
& 34+43=(3+4) * 11=7 * 11=77 \\
& 25+52=(2+5)^{*} 11=7 * 11=77 \\
& 81+18=(8+1) * 11=9 * 11=99 \\
& 75+57=(7+5)^{*} 11=12 * 11=132 \\
& 49+94=(4+9)^{*} 11=13 * 11=143 \\
& 44+44=(4+4) * 11=8 * 11=88 \\
& 73+37=(7+3) * 11=10 * 11=110
\end{aligned}
$$

The resulti s always given by the sum of the digits of the number multiplied by 11 .
We can call the pattern: starring at the 11!!

$$
\begin{aligned}
& 56+65=121 \\
& 87+78=165 \\
& 10+01=11 \\
& 99+99=198 \\
& 100+010=110
\end{aligned}
$$

3-digit mirror numbers:

$$
\begin{aligned}
& 123+321=444=(1+3) * 111=4^{*} 111 \\
& 234+432=666=(2+4) * 111=6 * 111 \\
& 345+543=888=(3+5) * 111=8^{*} 111 \\
& 456+654=1110=(4+6) * 111=10 * 111 \\
& 567+765=1332=(5+7) * 111=12 * 111
\end{aligned}
$$

$$
215+512=727
$$

$$
306+603=909
$$

$$
146+641=787
$$

$$
308+803=1111
$$

$$
419+914=1333
$$

## c) $\mathbf{2} / \mathbf{3}$ digit Mirror Numbers Multiplication

What happens in the multiplication?

$$
\begin{gathered}
32 * 23=736 \\
45 * 54=2430 \\
21 * 12=252
\end{gathered}
$$

For now it is not clear the pattern.

## D )2/3 digit Mirror Numbers Division

What happens in the division?

$$
\begin{gathered}
32 / 23=1,39 \\
54 / 45=1,2 \\
21 / 12=1,75
\end{gathered}
$$

For now it is not clear the pattern.

## Conclusions

VM helps us to use and to look for the solution to a problem in a simple, intuitive and flexible way.
This little discovery on the properties of 9 , and as a corollary also on 11 , are proof.
I invite you to help me to continue in search of patterns that can be useful in calculations of daily life, I assure you it will be a search and a discovery especially of ourselves!

## Link

Modular arithmetic: https://en.wikipedia.org/wiki/Modular_arithmetic
Giuliano Mandotti: http://giulianomandotti.com \& http://matematicavedica.it

