THREE BOOKS AND A PAPER: A GEOMETRY PROJECT

Andrew Nicholas

ABSTRACT

The aim of the project was to produce a sort of miniature Vedic mathematics version of Euclid's 'Elements', covering much less ground than the 'Elements' and doing so with greater speed and ease than Euclid achieved.

This task occupies three books; they are: 'Geometry for an Oral Tradition', 'The Circle Revelation' & 'Eight Essays on Geometry for an Oral Tradition'. This last book will hopefully be available later in 2016, free online

The paper 'A Much-needed Innovation in Geometry' is included because it shows how a Vedic approach helped formulate the foundations of 'Geometry for an Oral Tradition'.

A difficulty in the project was that for some one and a half centuries it has been known that the foundations of Euclid's 'Elements' are flawed. Putting this right is a major contribution of the project.

Reformulation of the foundations of geometry is done in two stages. First there is the 'Geometry for an Oral Tradition' (provisional) version, which makes use of a single axiom where Euclid needed ten.

Then, in the 'Eight Essays on Geometry for an Oral Tradition', the sole 'Geometry for an Oral Tradition' axiom is proved using two principles. The result is demonstrably rock-solid foundations for geometry.

NOTE The book 'Eight Essays on Geometry for an Oral Tradition' will hopefully be available later in 2016, free online

KEYWORDS: GEOMETRY, VEDIC MATHEMATICS, FOUNDATIONS OF GEOMETRY

The Project is to produce a much-reduced Vedic mathematics version of Euclid's ***Elements***, covering far less ground than Euclid does and doing so with greater speed and efficiency.

Three books contribute to the Project; what they each contribute is explained in the present paper.

The first book, ***Geometry for an Oral Tradition***, states and proves theorems as far as elementary properties of a circle. Its style is close to Euclid's in his book the ***Elements***, making it easier to relate to the ***Elements***. The drawback is that the formality of presentation makes both for a harder and a slower job in following it.

The second book, ***The Circle Revelation***, re-presents the material of ***Geometry for an Oral Tradition*** informally. It is written for schoolchildren. An adult can read this material in a few hours - little enough time, since the book presents a short course in geometry!

There are a few points to be made before explaining the contribution of the third book.

In the second half of the 19th century it became apparent that Euclid's foundations are inadequate. The ***Elements*** is based on ten axioms, which traditionally had been accepted on the grounds of being self-evident. But this consensus fell apart when it was discovered that being apparently self-evident is no guarantee of the truth of a statement.

The status of Euclid's axioms plummeted to being assumptions, generally (more on this later).

It cannot be acceptable to base a fundamental subject such as geometry on a set of assumptions.

The first step in sorting this matter out is taken in ***Geometry for an Oral Tradition,*** which, instead of Euclid's ten axioms relies on a single axiom, namely, ***magnitudes are unchanged by motion***. (The term **postulate** is used in ***Geometry for an Oral Tradition,*** rather than axiom, a postulate being defined there as an assumption made for the purpose of the study.) However, relying on even one assumption fails to set the subject on terra firma.

The third book, ***Eight Essays on Geometry for an Oral Tradition***, eliminates this single axiom (postulate) by using two well-supported principles to prove it, as we show next. The result is rock-solid foundations.

PROOF OF THE *GEOMETRY FOR AN ORAL TRADITION* POSTULATE

To be proved. Magnitudes are unchanged by motion.

Given.
Principle 1 Once completed, a figure in a plane at rest remains unchanged.
Principle 2 If any two planes are in relative motion, either of them can serve as our standard of rest. That is to say, all states of rest and motion are relative.

Proof

Calling the two relatively moving planes A and B, let us first treat
plane A as our standard of rest. [By Principle 2]
Then a figure drawn in plane A remains unchanged [by Principle 1],
and so do its magnitudes.
Now, instead, let plane B serve as our standard of rest.
Then plane A is in motion relative to plane B,
But we have just shown that the magnitudes in plane A remain unchanged.
Therefore the motion of the magnitudes in plane A relative to plane B
does not change them.
That is, magnitudes are unchanged by motion. QED.

Now we come to a question.

WHY HAVE SOUND FOUNDATIONS NOT BEEN FOUND BEFORE?

Judging by attempts which have been made to sort the matter out, the belief has been that, since Euclid's axioms are unsatisfactory the need is to find more satisfactory axioms. This amounts to replacing one set of assumptions by another set of assumptions. Rock-solid foundations are not obtainable in this way. (An attempt to provide more satisfactory axioms is discussed in Ref. 5.)

Let us take a closer look at some of Euclid's axioms, first covering ground also useful to us in another way as we do so.

GROUNDS FOR ACCEPTING A STATEMENT

1) It has been proved.

2) There is adequate evidence for it.

3) It is self-evident, and accepted as an axiom.

4) It is an assumption.

Re the second point above, consider the first three of Euclid's postulates. They are as follows.

Let the following be postulated:

Postulate 1. To draw a straight line from any point to any point.
Postulate 2. To produce a finite straight line continuously in a straight line.
Postulate3. To describe a circle with any centre and distance.

Euclid does not say that he is using a straight edge and compasses; however, these first three postulates refer to what you can do with them, as anyone familiar with these instruments will be aware. What is being relied on here is ***evidence*** obtained from use of a straight edge and compasses.

Euclid gives two more postulates and five common notions (note that the *Oxford World Encyclopedia* calls these *postulates* and *assumptions* respectively). These seven are not supported by evidence, and so, since being self-evident is no longer considered to be grounds for accepting a statement we are obliged to treat them as assumptions. That is to say, instead of being soundly based Euclid's *Elements* rests on seven assumptions (plus three postulates which are well-supported by evidence). (Essay 7 of the *Eight Essays On Geometry for an Oral Tradition* considers this in more detail.)

The other point covered by the above analysis is that being adequately supported by evidence is grounds for accepting a statement, and *this applies to Principles 1 and 2 used in the proof given above.* (The evidence is spelt out in Essay 6 of the *Eight Essays On Geometry for an Oral Tradition*.)

So much for the three books of the title; now for the paper.

WHAT THE PAPER *A MUCH-NEEDED INNOVATION IN GEOMETRY* CONTRIBUTES

It explains how a Vedic mathematics approach proved helpful in formulating the foundations for *Geometry for an Oral Tradition,* in various ways. These include:

1) Having no more than a handful of geometry proofs initially (from Tirthaji's book *Vedic Mathematics*), a completely fresh start for the study of geometry was needed.

2) The paper explains why there was no reason to suppose that axioms are needed in Vedic geometry.

3) The Vedic tradition is an oral one, which draws attention to the vital role of language and especially of speech in the study. (This is discussed further in the paper *A Much-needed Innovation in Geometry*, and also in *Geometry for an Oral Tradition*, both in the Preliminaries and in the Commentary, Part II, Section 3(f).)

4) The word 'Innovation' in the title of the paper refers to treating determination of the foundations as a problem to be solved. Euclid did not do this and there is no reason to think that anyone else has thought of doing so prior to the present project. The Vedic approach has made a contribution here, too, as the above-mentioned 'Innovation' paper explains.

Note that this 'problem to be solved' approach was crucial to eventual success. (See Essays 3 and 7 of the *Eight Essays on Geometry for an Oral Tradition* for an explanation of how the foundations were worked out.)

* * * * * * * * * * *

Finally, two matters conclude the present paper; they are:

(i) A statement of the foundations of geometry.
(ii) An example of Euclid's proof of one of his theorems, and the brief proof of the same theorem given in both *The Circle Revelation* and ***Geometry for an Oral Tradition.***

(i) THE FOUNDATIONS OF GEOMETRY

These consist of the two Principles given above, plus the first three Provisions given in *Geometry for an Oral Tradition*. The three Provisions can readily be seen to be indispensable for the study: for remove any one of them and the study is not possible.

PROVISIONS

1. **A language, in use.**
2. **A means of drawing figures in a plane, using a straight edge and compasses.**
3. **The ability to recognise valid reasoning.**

PRINCIPLES

1. **Once completed, a figure drawn in a plane at rest remains unchanged.**

2. **If any two planes are in relative motion, either may be used as the standard of rest. That is to say, all states of rest and motion are relative.**

There are a few further points to be made concerning these foundations.

1. Euclid takes Principle 1 for granted without mentioning it, and *Geometry for an Oral Tradition* likewise.

2. Principle 2 is necessary because it is well-established that space is relative. In combination with Principle 1, Principle 2 also meets the need to start the sequence of Propositions off, via proof of the postulate used in *Geometry for an Oral Tradition.*

3. Thus Principles 1 and 2 are necessary for the study, and so are the three Provisions (the indispensables). To establish that the foundations are both NECESSARY and SUFFICIENT for the study, we now need to show to what extent they suffice. They certainly suffice to take the sequence of propositions at least as far as elementary properties of s circle, as *Geometry for an Oral Tradition* shows; we cannot rule out that they suffice to take the investigation much further.

(ii) TWO PROOFS OF A THEOREM

Euclid' proof of Book I Proposition 32 is given in an attachment. It shows that the angles in a triangle total two right angles. In *Geometry for an Oral Tradition* the same theorem is covered by Proposition B17, which states that the angles in a triangle total a half turn. The proof is given in a second attachment; *The Circle Revelation* re-presents this proof in a more informal way.

REFERENCES

1. Sir T L Heath, The Thirteen Books of Euclid's ***Elements*** Cambridge University Press, 1908
2. A P Nicholas, ***Geometry for an Oral Tradition*** Inspiration Books, 1999
3. A P Nicholas, ***The Circle Revelation*** Inspiration Books, 1999
4. A P Nicholas, ***Eight Essays on Geometry for an Oral Tradition*** NOT YET PUBLISHED
5. A P Nicholas, ***Guess Which Subject Lacks Sound Foundations,* Vedic Maths Newsletter Issues 43 & 44, February 2005 and April 2005**
6. ***The Oxford World Encyclopedia***, Oxford University Press, 1998
7. A P Nicholas, ***A Much-needed Innovation in Geometry*** NOT YET PUBLISHED
8. B K Tirthaji, ***Vedic Mathematics***, Motilal Banarsidass, 1965